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NONLINEAR DISTORTION IN THE PROPAGATION OF INTENSE ACOUSTIC NOISE

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As sound pressure increases to "finite" levels, nonlinear effects usually ignored in acoustics problems become increasingly important. In this paper plane waves propagating through a pipe are considered. In the first section the distortion of a wave that is sinusoidal at the source is treated. In the second section the source signal is a pulse of band-limited (500 to 3500 Hz) random noise. In both cases theoretical results are compared with experimental measurements. Extension of the analysis to the more practical case of outdoor propagation is indicated.

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Abstract

As sound pressure increases to "finite" levels, nonlinear effects usually ignored in acoustics problems become increasingly important. In this paper plane waves propagating through a pipe are considered. In the first section the distortion of a wave that is sinusoidal at the source is treated. In the second section the source signal is a pulse of bandlimited (500 to 3500 Hz) random noise. In both cases theoretical results are compared with experimental measurements. Extension of the analysis to the more practical case of outdoor propagation is indicated.

A good deal of fundamental research, both experimental and theoretical, has been done on finite amplitude* waves. Almost all of it has been done on deterministic waveforms; very little has been done on finite amplitude noise.[1,2] The ultimate goal of the present research is to attack the problem of propagation of finite-amplitude noise. In an effort to simplify the problem as much as possible, it was decided to restrict the investigation to plane waves. But how does one handle a finite amplitude wave whose waveform is not deterministic? Our approach has been to use weak-shock theory, cast in the form of a computer algorithm, to obtain theoretical predictions and to make measurements in a plane wave tube to check these predictions. To test this approach we have made a preliminary study of initially sinusoidal waves. This has been followed by a study of the propagation of pulses of random noise. The noise is Gaussian and bandlimited to approximately 3 octaves. Sound pressure levels up to 160 dB (re 0.0002 μbar) have been used. The results of these studies form the basis of this paper.

Use of the plane wave tube has introduced the complication of tube wall attenuation and dispersion. Accordingly, we have had to modify weak-shock theory to take account of these effects. It is simpler in the beginning, however, to consider distortion in the absence of tube wall effects.

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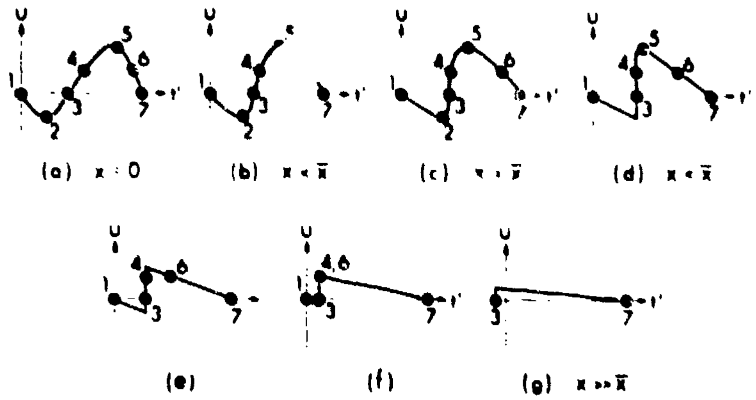
*"Finite amplitude" is a qualitative term. In general it implies that nonlinear effects, which are ignored in linear acoustics, must be accounted for. However, some distance may be required for the nonlinear effects to be noticeable. For example, in air an initially sinusoidal plane wave of SPL 130 dB (re 0.0002 μbar) forms shocks at a distance of about 210 wavelengths. An observer much closer than this to the source would probably view the wave as "infinitesimal" whereas an observer much further away would see clear evidence that the wave is of "finite amplitude."

When sound waves of large amplitude propagate, the nonlinearity of the propagation process causes progressive distortion of the waveform. In a nondispersive medium this distortion will cause a sine wave to deform into a sawtooth wave, provided, of course, the wave does not first become "infinitesimal" through absorption. Figure 1 depicts this progressive distortion for an arbitrarily shaped wave. The vertical axis is particle velocity (or pressure) and the time base is retarded time $t'=t-x/c_o$. Note how the portions of the waveform having larger particle velocities tend to overtake the slower velocity portions. When this overtaking actually occurs, shocks form. This is clearly shown at $x=\bar{x}$.



CONTINUOUS: $\frac{dt}{dx} = (c_0+\beta u)^{-1} \doteq c_0^{-1}(1-\frac{\beta u}{c_0})$

SHOCK: $(\frac{dt}{dx})_s \doteq c_0^{-1}\left[1-\frac{\beta(u_a+u_b)}{2c_0}\right]$

FIGURE 1
WEAK-SHOCK THEORY

Mathematically, the progressive distortion of the continuous portion of the waveform is described by the propagation law:

$$\frac{dt}{dx} = \left[c_o+\beta u\right]^{-1} \doteq c_o^{-1}\left[1-\frac{\beta u}{c_o}\right] \quad . \tag{1}$$

Here x is distance, t time, u particle velocity and c small-signal sound speed; for gases $\beta = \frac{\gamma+1}{2}$ (γ is in the ratio of specific heats) and for liquids $\beta = \frac{B}{2A}$ (B/A is a nonlinearity parameter). At the discontinuity the propagation of the shock front is described by the weak-shock relation

$$\left(\frac{dt}{dx}\right)_s \doteq c_o^{-1}\left[1-\frac{\beta\left(u_a+u_b\right)}{2c_o}\right] \quad , \tag{2}$$

where u_a and u_b are values of u just ahead of and just behind the shock, respectively. Notice how different points on the waveform move with respect to each other. As dissipation at the shock front begins to take its toll of the wave, points of the waveform in the neighborhood of the shock just disappear. In the end only those sections of the waveform that were originally near the zero crossings survive.

Although an explicit analytical solution of Eqs. (1) and (2) (and appropriate auxiliary equation) can be found for simple waveforms, such a solution is not feasible for complicated waveforms, particularly non-deterministic ones such as noise. A computer algorithm in which Eqs. (1) and (2) are used as the basis for calculating the distortion of an arbitrary waveform is, however, possible.

We have developed such an algorithm. Equations (1) and (2) are cast in the form of difference equations and solved. The wave is "computer propagated" a small distance x, and the original time base is distorted in accordance with the relation

$$t'_{new} = t'_{old} - \beta u_{old}\, x/c_o^2 \quad . \tag{3}$$

This distorted time base is scanned for multivaluedness. If none is found, another small distance step is taken. This sequence is repeated until finally multivaluedness, which signifies the presence of a shock, is found. The shock is located as prescribed by Eq. (2) and the particle velocities at the shock front are adjusted accordingly. In a periodic waveform, such as a sawtooth wave, the shock fronts are stationary in the retarded time frame because the shocks move with speed c_o. However, in an arbitrary wave field the shocks exhibit relative motion. A separate portion of the distortion program monitors this motion and accounts for shock merging. Through the progression of growing, decaying, and merging, the shocks ultimately determine the shape and amplitude of the wave. The distortion algorithm has been tried on several initial signals, including N waves, sinusoids, and randomly generated waves. The results are in excellent agreement with such analytical results as are known.

Unfortunately, this simple algorithm does not give a very good account of itself when the wave motion is in a tube. In 1965, McKittrick, Blackstock, and Wright[3] reported on the measurement of the wave shape of repeated shocks in a tube. Rather than exhibiting the sawtooth shape that was expected, these waves had a markedly rounded top while retaining a sharp trough. It was demonstrated that the asymmetry was due to dispersion caused by tube wall effects. In a theoretical study Coppens[4] confirmed the asymmetry and its cause (dispersion) for waves in the pre-shock region. We have also observed the same asymmetric waveform in our measurements. It should be noted that the tube wall causes attenuation as well as dispersion.

Our method of accounting for tube wall effects is to incorporate attenuation and dispersion directly in our distortion algorithm.* See Fig. 2. Briefly, for each incremental propagation step the wave is appropriately distorted. Then an FFT routine is used to express the wave in terms of its spectral components. The complex amplitude C_n of each spectral component is corrected for the attenuation and phase shift that occurs over the incremental step as follows:

$$C_n' = C_n \, e^{-\sqrt{n}\,\alpha x(1+j)} \quad , \tag{4}$$

where C_n' is the corrected amplitude and α is the attenuation coefficient for the fundamental. As is well known for tube wall attenuation α is proportional to $\sqrt{\omega}$, and this is why the factor $\sqrt{n}$ appears for the n^{th} harmonic. Note also the phase shift, which is caused by the dispersion. Since the phase shift is different for each harmonic, the spectral components get slightly out of step with each other. This is what causes the symmetry of the waveform to be ruined, particularly at the shocks. The asymmetry has an effect on the overall decay of the wave, as will presently be explained. The waveform is inverse transformed and another distance step is taken. In this way, the wave profile at any distance from the source is realized.

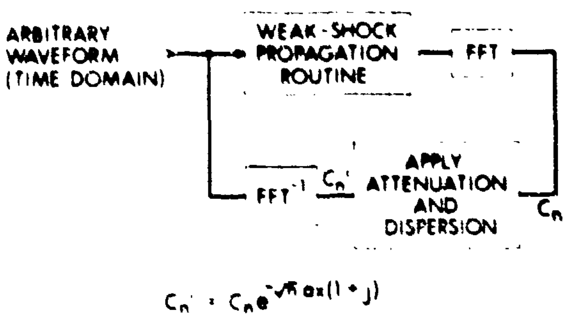


FIGURE 2
A SCHEMATIC DIAGRAM OF
MODIFIED WEAK-SHOCK THEORY

Now let us turn to the experiment. The plane wave tube is a 96 ft, 2 in. i.d. aluminum pipe composed of eight 12 ft pipe sections. The sections are joined by carefully constructed 2-piece flanges that are designed to minimize reflections at the joints. The pipe is terminated in a suitable nonreflecting fiberglass wedge. Source frequencies in the range 800 to 3500 kHz and SPL's in the range 140 to 160 dB have been used. In the following results the fundamental frequency was 2 kHz and the initial SPL was 160 dB. The sound source was an ALTEC 290 E "Giant Voice"

*The algorithm thus becomes similar to one used by Cook[5] to calculate the distortion of a sinusoidal wave in a free thermoviscous medium. Cook did not, however, account for shocks in the waveform by means of weak-shock theory.

horn driver powered in a pulsed mode by a 200 W Dukane power amplifier. The pickup microphone was a B & K 1/4 in. type 4136, positioned in a mount so that the diaphragm was flush with the pipe wall. The microphone output was displayed on an oscilloscope and the waveform photographed. Figure 3 shows a generalized diagram of the equipment.

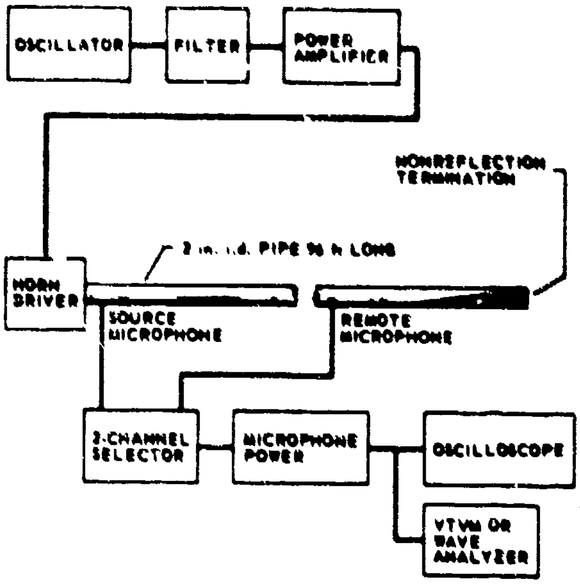
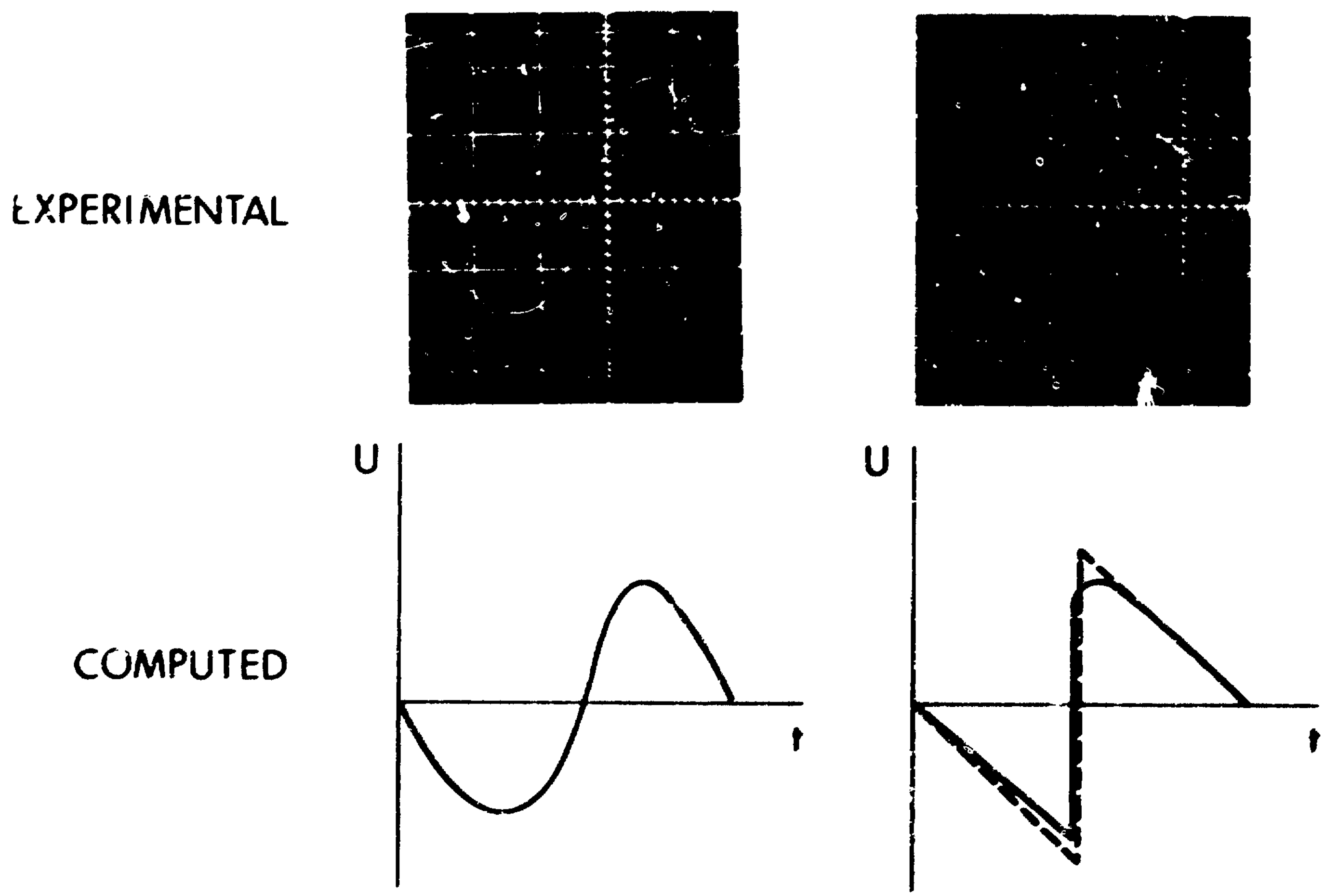


FIGURE 3
A GENERALIZED EQUIPMENT ARRANGEMENT

Figure 4 shows one cycle of the measured and computed waveforms at distances of 1 ft and 13 ft from the sound source. The waveform at one foot is already somewhat distorted primarily because of nonlinearities in our sound source. This waveform is the input signal for both the experiment and the computations. At 13 ft the wave has traveled about 3 shock formation lengths so that on the basis of simple weak-shock theory it would be expected to be essentially perfect sawtooth. Because of wall effects, however, there is already some rounding at its peak. Notice that only the peak is rounded, not both the peak and the trough. As explained earlier, the asymmetry is due to dispersion. The dashed waveform was calculated by omitting the wall effects portion of the algorithm. Thus, this is the waveform one would expect in an open medium and, of course, is the same one could compute analytically from ordinary weak-shock theory.



x = 1 ft
VERTICAL AMP. = 1

x = 13 ft
VERTICAL AMP. = 2

FIGURE 4

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AS-72-1459e-S
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11 - 17 - 72

In Fig. 5 the 13 ft comparison has been repeated and experimental and computed waveforms at 49 ft and 85 ft added. Note the increased asymmetry, particularly the rounding of the peak, as the wave propagates. Although the wave depicted here is a relatively strong one, we have used our method to treat weaker waves with equally good results. The difference between the solid and dashed line theoretical curves shows how important tube wall effects are under our experimental conditions.

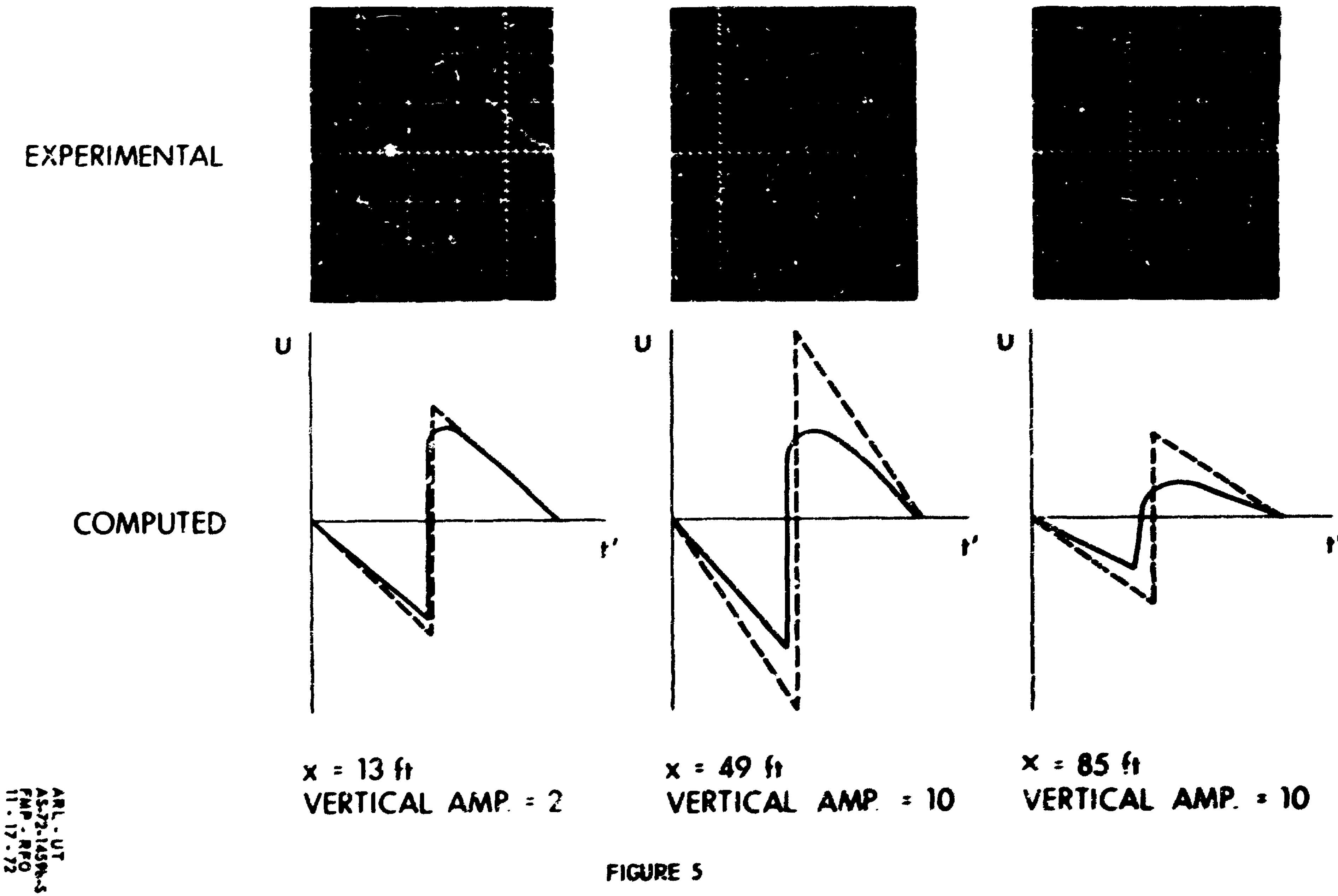
In Fig. 6 three theoretical curves of the decay of the wave with distance are shown together with our experimental results. The top curve is for ordinary weak-shock theory. Again, this is the same as our computer method gives if the effects of the tube wall are omitted entirely. The only attenuation is that associated with the dissipation taking place at the shocks. The middle curve is obtained from our computed waveforms and thus represents a model in which tube wall attenuation and dispersion are included along with shock dissipation. The lowest curve is for weak-shock theory corrected for tube wall attenuation but not for dispersion. To obtain this curve we have used the equation first suggested by Rudnick,[6]

$$\frac{dp_{max}}{dx} = - \alpha p_{max} - \frac{\beta \epsilon k}{\pi p_o} p_{max}^2 \quad . \tag{5}$$

This equation gives the rate of decay of a sawtooth wave as the sum of the decay due to ordinary attenuation (the first term) and the decay due to dissipation at the shocks (the second term). Here p_{max} is peak pressure, x distance, $\epsilon = u_o/c_o$, k wave number, p_o the peak pressure at x=0, and β as defined previously. The value of α is given by Rudnick's summation method for calculating the effective tube wall attenuation coefficient.

The experimental points confirm the middle curve. The top curve represents a prediction of too little attenuation because tube wall absorption is neglected. Yet when wall absorption is accounted for, one obtains the lowest curve, and too much attenuation is predicted. We believe that the key to the success of our model, i.e., the middle curve, is the dispersion. The severe rounding of the peaks of the shocks caused by dispersion in effect reduces the amplitude of each shock. Instead of extending from trough to peak, the actual shock runs only from trough to a point short of the peak. This lessens the shock dissipation. In other words, dispersion inhibits the attenuation associated with the shock. When weak-shock theory is modified to take account of both dispersion and attenuation, the result is the middle curve.

The decay of sawtooth waves in tubes has received the attention of several previous investigators.[6,7,8] In general they found a discrepancy between their measured values of decay and predicted values obtained by adding wall losses to shock dissipation. As we now know, dispersion should have been taken into account as well as wall attenuation. However, when we compared their data with predictions based on our theory, in which account is taken of dispersion, completely satisfactory agreement


EXPERIMENTAL
COMPUTED
U
t'
x = 13 ft
VERTICAL AMP. = 2
x = 49 ft
VERTICAL AMP. = 10
x = 85 ft
VERTICAL AMP. = 10
ARL - UT
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11 - 17 - 72

FIGURE 5

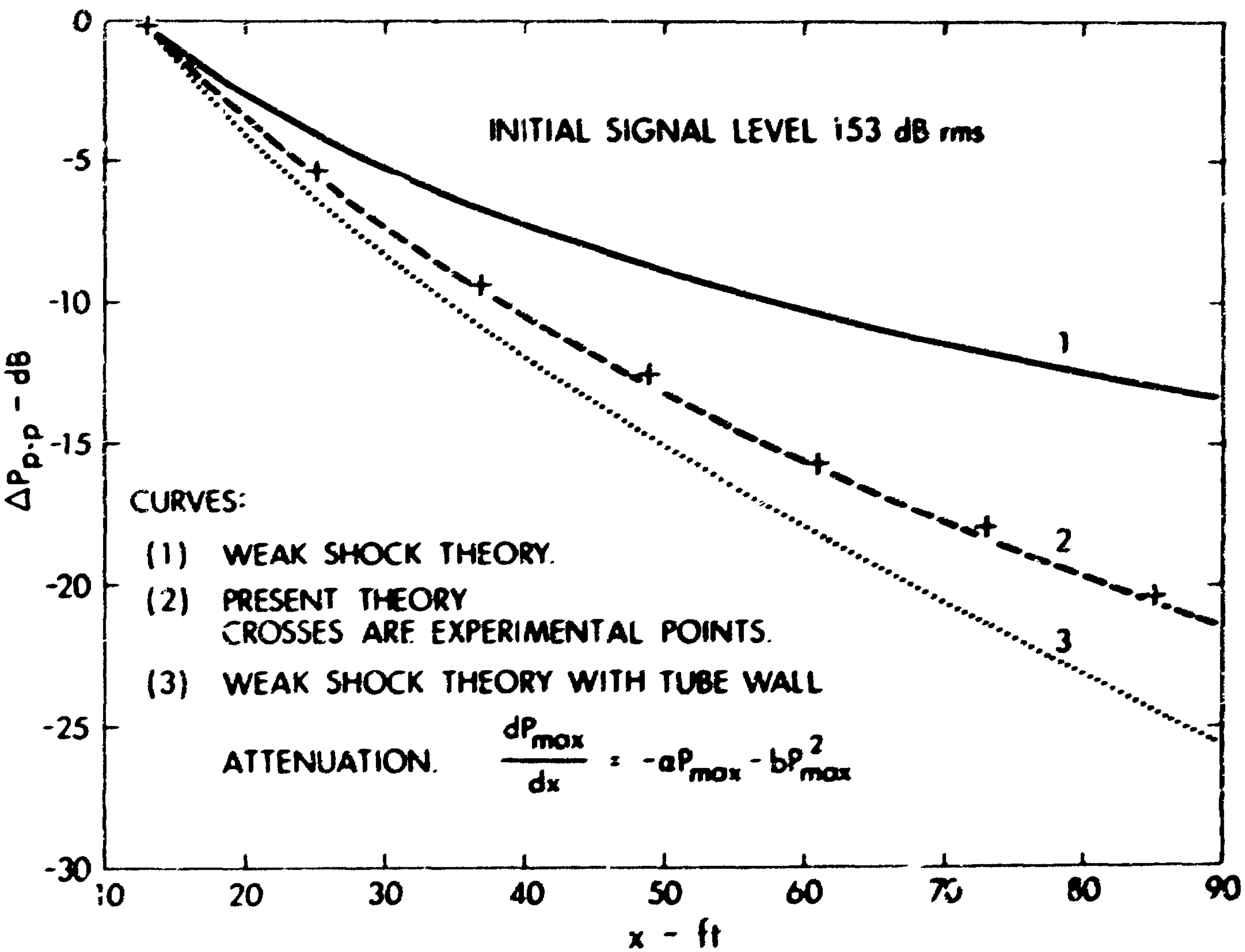

FIGURE 6
20 $\log_{10} \Delta P$ vs DISTANCE

was still lacking. The reasons for this are not clear, in that we have found good agreement with our own measurements for a variety of frequencies in the range 800 to 3500 Hz and sound pressure levels in the range 140 to 160 dB. There are some differences between the earlier measurements and ours. We have generally used higher frequencies and somewhat lower sound pressure levels. Further, the emphasis in the earlier work was on peak-to-peak amplitude. Little attention was paid to wave shape so that we do not know the extent to which rounding of the peaks of the waveform (Figs. 4 and 5) might have influenced interpretation of the data. Under the circumstances, all that can be said definitely at this point is that our theoretical method has been verified for the middle audio ranges for sound pressure levels up to about 160 dB.

In order to use the previously described algorithm for noise pulses, only a few program changes are necessary. The fundamemtal frequency becomes

$$f_{fund} = 1/T \quad , \qquad (6)$$

where T is the length of the pulse. Attenuation and dispersion are again applied to each spectral component up to the Nyquist frequency.

The experimental measurements were made on a particular pulse of noise as follows: First a random electrical pulse of noise was bandpass filtered and recorded on a tape loop. The pulse was then played back, amplified, and applied to the sound source, in this case a University 65 W horn driver. As a result of the filtering action of the electrical filter and the horn driver, the spectrum of the acoustic noise pulse as it started down the tube extended from about 500 Hz to about 3500 Hz. The initial (pulse) SPL was 160 dB, and the pulse repetition rate was one per second. A memory type oscilloscope was used to record the waveform of the pulse at various distances down the pipe.

In Fig. 7 the experimental and computed waveforms are compared. Note how the noise signal simplifies as the wave propagates.

The number of zero crossings diminishes and minor irregularities in the waveform disappear as they are "eaten up" by the larger shocks. Note also the rounding of the wave peaks. As in the sinusoidal case, this is caused by dispersion. It will be seen that the agreement between theory and experiment is quite good.

In terms of frequency, the reduction in number of zero crossings represents a transfer of energy to the lower end of the spectrum. On the other hand, the presence of the shocks with their very fast rise times means that the high frequency end of the spectrum has also been enhanced. In general the relative buildup of the upper and lower ends of the spectrum takes place at the expense of the middle of the spectrum. The effect of nonlinearity is thus to flatten out the spectrum.

In Fig. 8, the 13 ft and 85 ft computed waveforms from Fig. 7 are repeated and compared to waveforms computed by using the algorithm based

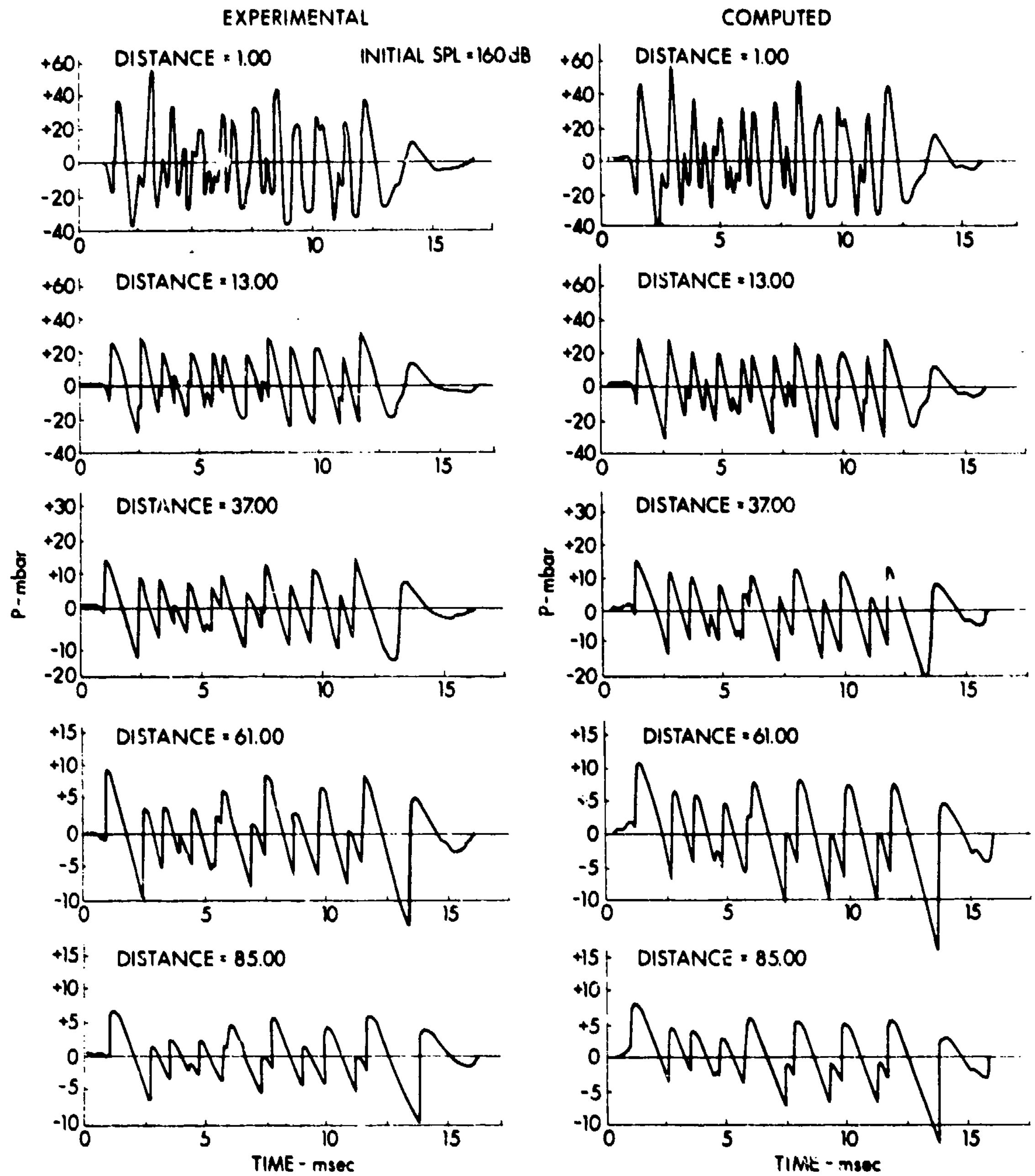

FIGURE 7
NOISE PULSE AT VARIOUS DISTANCES

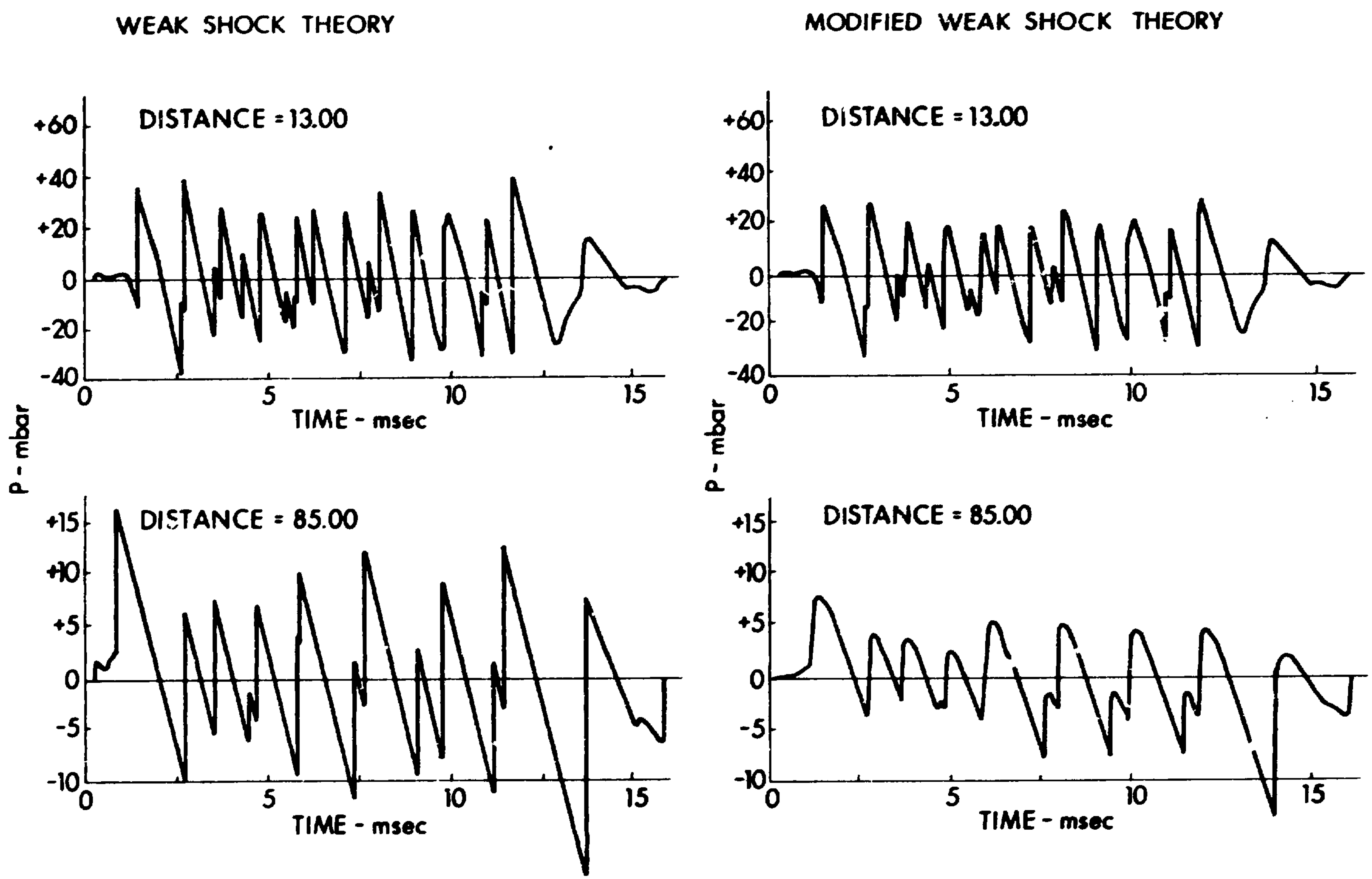

FIGURE 8

A COMPARISON OF WEAK SHOCK
AND WEAK SHOCK THEORY MODIFIED BY
TUBE WALL EFFECTS

on weak-shock theory alone. In the latter case, no correction was made for attenuation and dispersion caused by wall effects. It is clear that the wall effects have a significant influence on the waveform, particularly at the greater distance.

In conclusion, we feel that our model adequately describes the propagation of plane sound waves in a real pipe in the frequency range of at least 0.5 to 3.5 kHz, at sound pressure levels up to at least 160 dB. The application of the model to outdoor propagation is expected to be straightforward. The transformation from plane to spherical or cylindrical waves is well known.[9] Furthermore, the absence of wall effects will represent a simplification.

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